COMPUTER SCIENCE TRIPOS Part IA – 2024 – Paper 1

2 Foundations of Computer Science (avsm2)

(a) A prime number is a natural number greater than 1 that has no positive divisors other than 1 and itself. We wish to implement a primality test in OCaml that checks if a positive input integer is prime. A simple primality test is via trial division: given an positive input number n, check if it is divisible by any prime number between 2 and \sqrt{n} . For any divisor $p \ge \sqrt{n}$, there must be another divisor $n/p \le \sqrt{n}$, and a prime divisor q of n/p, and therefore looking for prime divisors where $p \le \sqrt{n}$ is sufficient.

Define a function is_prime which accepts a positive input integer and returns a boolean to indicate if it is prime or not. To simplify your code, you can avoid calculating square roots by checking for prime divisors where $p^2 \leq n$. You can assume the existence of a (mod) operator which returns the integer remainder of two integers. For example, 3 mod 2 will return 1. The type definitions are:

val (mod): int -> int -> int val is_prime : int -> bool

[8 marks]

- (b) In functional programming, *fold* functions are higher order functions that process data structure elements in order and build a return value.
 - (i) fold_range a b f acc is a specialised integer fold that applies f(n) where b ≤ n ≤ a, with initial value acc. For example, fold_range 1 3 (+) 10 would return 16 (from 10+1+2+3). Define this function with the type:

val fold_range: int -> int -> (int -> 'a -> 'a) -> 'a -> 'a [4 marks]

(ii) fold f acc l is a more generic function that applies f over a list l with initial value acc. Define this function with the type:

val fold: ('a -> 'b -> 'a) -> 'a -> 'b list -> 'a

[2 marks]

- (*iii*) Explain the time and space complexity of your implementation of fold and briefly discuss whether it is tail recursive or not. [3 marks]
- (c) all_primes a b is a function that returns all of the valid prime numbers between a and b inclusively. Define this function and its associated type, using the previous definitions of fold_range and is_prime.
 [3 marks]